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ME 518

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Planetary Differential Gearset Term Project Report

1. Abstract

The objective of this term project was to create and analyze a differential planetary gear train system in the Adams simulation software. MATLAB scripts provided by Dr. Wu were used to create gear teeth involute profiles which were imported into SolidWorks to create the individual gear CAD models. The created gear CAD models were assembled in SolidWorks and exported into the Adams simulation software. Various joints, contacts, motions and forces were created within Adams to kinematically constrain the gearset during simulation. Hand calculations were performed via a MATLAB script to verify the accuracy of the Adams simulation after which the simulation was ran under several different load cases. Both ideal and cracked planetary gearsets were models and simulated in Adams, however there were no noticeable differences in the simulation results. Through the completion of this project several insights were learned about both planetary gearboxes and Adams simulations.

2. Introduction

A planetary gear system is a type of gear box which is composed of the following four main components: sun gear, planet gears, ring gear and carrier. The sun gear is usually the smallest of the components and sits in the center of the gear system. The ring gear forms the perimeter of the gear train and is cut with internal teeth. Finally, the planet gears revolve around the three spokes of the carrier which in turn revolves about the center of the gear set. Planetary gearboxes are unique in that the planet gears can both rotate about their center axes while translating in a circular arc about the center of the gearbox. This can allow for large gear reductions in a very compact package.

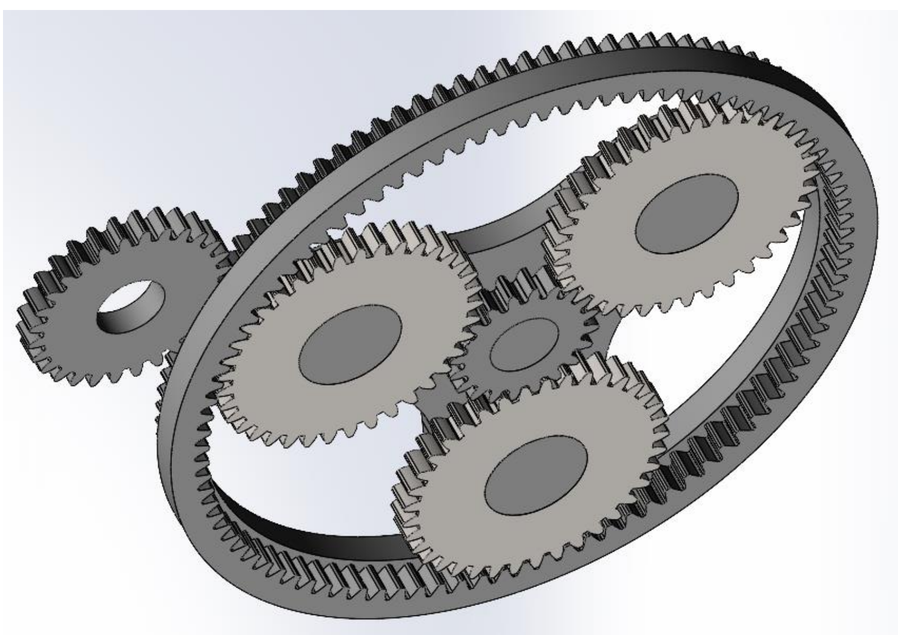


Figure 1 Cracked Planetary Gear Train Assembly CAD Model

In a normal planetary gearbox, the ring is fixed and there is one input to the system in the form of carrier or sun rotation. A differential planetary gearset frees up the rotation of the ring via the addition of another gear. In a differential planetary gearset, there are two inputs to the system, usually in the form of carrier, sun or peripheral gear angular rotation. *Figure 1* depicts the planetary gearset analyzed in this project.

3. SolidWorks Modeling

3.1 Generation of teeth profile

The modeling process began with creating the gear tooth profile. Gear teeth are shaped based on an involute profile which maintains a constant pressure angle throughout its rotation. The involute tooth profile is not an easy shape to create using the basic tools available in CAD software, so a MATLAB script provided by Dr. Wu was used to generate the profile outside of the CAD software. Based on the input number of gear tooth and gear module, the provided script calculates the tooth profile and then exports a text file containing the positional data of the curve. The script also calculates the gear's pitch circle, addendum and dedendum circles. *Figure 2* depicts the gear profile calculated by the MATLAB script along with the gear circles.

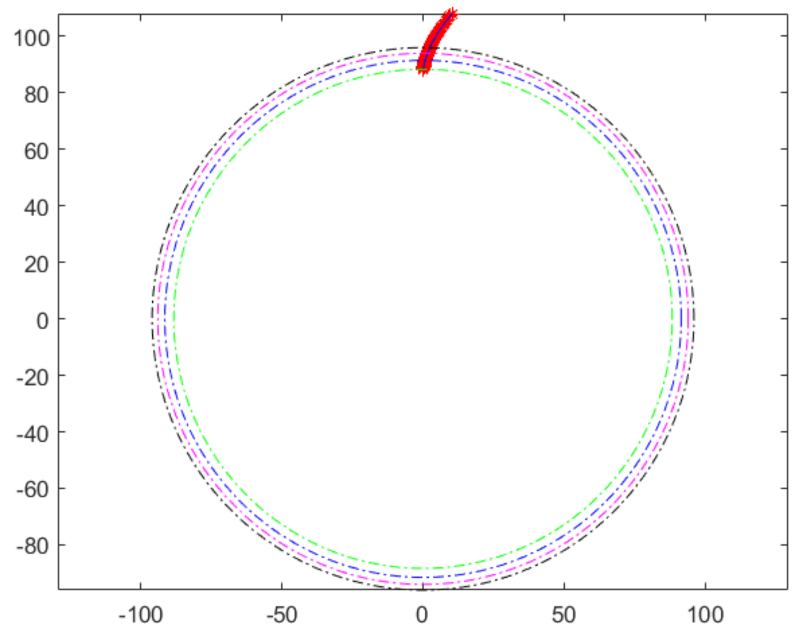


Figure 2 Example MATLAB gear tooth profile calculation

The script provided by Dr. Wu requires that the exported curves be manually shifted after importing them into the CAD software. Fortunately, Niko created an edited version of the script which will rotate the curves based on whether the gear is internal or external, and the desired percent backlash. Backlash is the play between gears in a gear train and is introduced by reducing the thickness of the gear teeth from their ideal shape by some factor. The gear teeth used in this project were designed to be 98% of their ideal width to ensure proper mating of gears during Adams simulations. By rotating the curves before exporting from MATLAB, the curves are already in the correct location when imported into the CAD models. See Appendix A for Niko's edited version of the script provided by Dr. Wu.

3.2 Solid modeling

After creating the shapes of the gear teeth in MATLAB, the curves were then imported into a CAD software, in this instance SolidWorks. The gear tooth profile was then connected to its addendum and dedendum circles and extruded to the specified depth. All gears used in this project are 8 millimeters thick except for the ring and Z2 gears which are 9 millimeters thick. This additional thickness is included to prevent rubbing along the face of the gear which can create problems during Adams simulations. Fillets were also added to the gear teeth to smooth out the gear meshing process. Note that to properly model the gears in SolidWorks, the modeling tolerance must be set to its finest level. See *Table 1* for a list of the gear parameters used in this project.

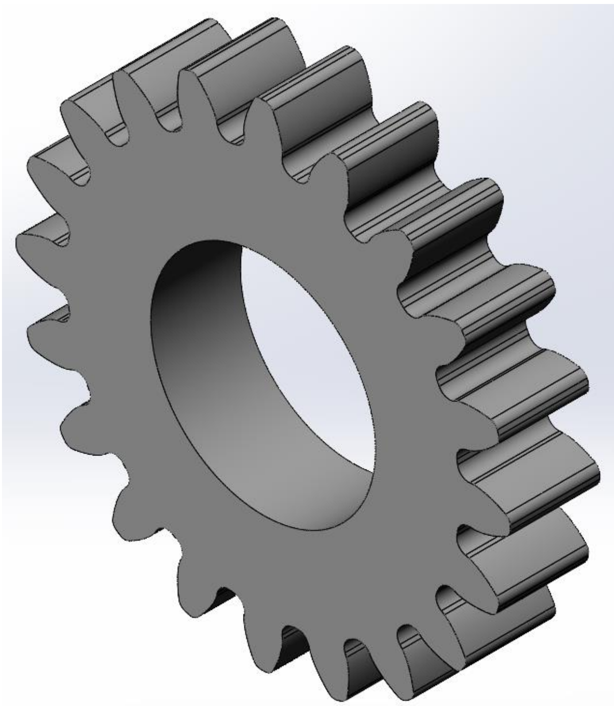


Figure 3.A Uncracked Sun Gear CAD Model

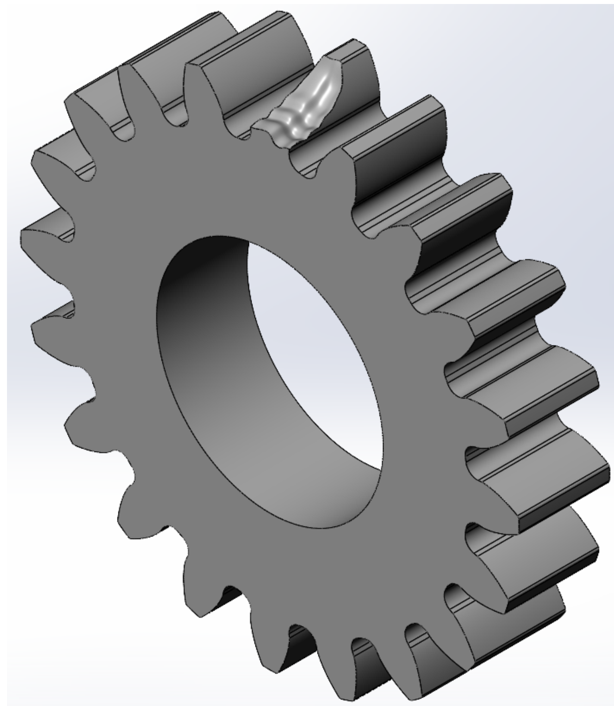


Figure 3.B Cracked Sun Gear CAD Model

After creating the individual gears, the sun gear was selected to be the damaged gear, since sun gears are usually the first to fail in real life gear trains. The gear shape was modified using a swept cut to better represent a chipped gear in real life. *Figure 3.A* and *Figure 3.B* compare the uncracked and cracked versions of the sun gear. See *Figure 1* for the complete SolidWorks assembly of the gear train.

Table 1 Number of teeth per gear and their associated colors and Adams

Gear	Number of teeth	Pressure Angle (degrees)	Thickness	Module	Adam's Color
Sun	20	20	8 mm	2 mm	Magenta
Planet	37	20	8 mm	2 mm	Red
Ring	94	20	9 mm	2 mm	Peach
Z1	28	20	9 mm	2 mm	Peach
Z2	98	20	8 mm	2 mm	Cyan

4. Adams Modeling

After creating the cracked and uncracked CAD models in SolidWorks, their Solid Bodies were exported as step files and imported into Adams. Prior to importing the files, the Adams modeling resolution was set to 0.01 to ensure that the imported bodies were not distorted. All solid bodies in Adams were modeled as rigid bodies, so the torsional and bending modes of the shafts are not taken into consideration. An isometric view of the Adams assembly is presented in *Figure 4*.

4.1 Model setup: Joints

As previously mentioned in the introduction section of this report, the orange carrier supports the three planetary gears, all of which rotate about the center sun gear. Furthermore, the ring gear of the NGW planetary gear drive used in this project also rotates about the center, based on the input speed from the Z2 gear. These connections were simulated in Adams using rigid body revolute joints. These joints have infinite radial stiffness and are equivalent to modeling the gear train with rigid bearings.

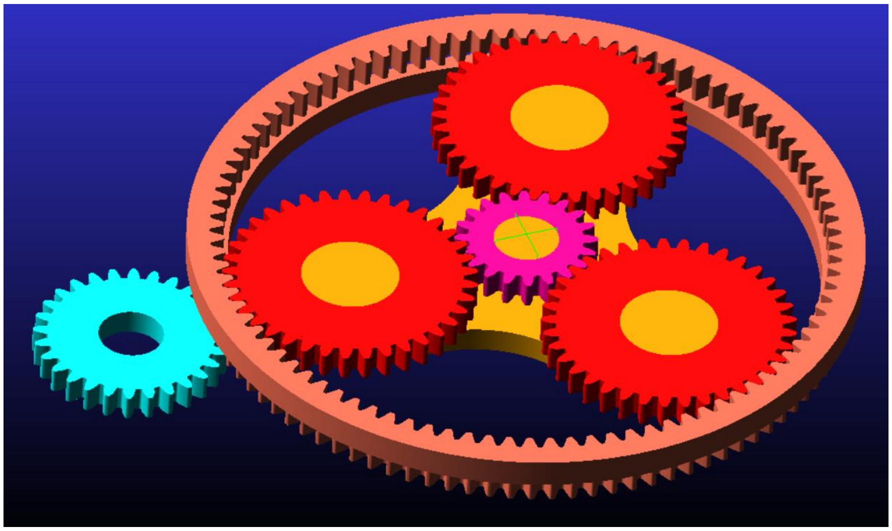


Figure 4 Cracked planetary gear train Adams model setup

While creating the joints, the geometric centers of the gears were selected to be the points of rotation. This detail is important for the cracked gear, as its center of mass and geometric center are not coincident. Using the eccentric center of mass as the center of rotation should be avoided as it will create erroneous results. Revolute joints were created to connect the planetary gears to the carrier while all other gears were connected via revolute joints to ground.

4.2 Model Setup: Contact Forces

The interaction of the gears with each other was modeled using contact forces between the gears. Contact forces allow for relatively accurate modeling of backlash and impact between gears, and the tuning of the contact parameters is a critical stage of the modeling process. Contact forces were created between the meshing gears, and no contact forces were applied to the carrier gear.

There are two types of contact modeling methods in Adams, the impact method and the restitution method. The impact method calculates the resultant forces, velocities and displacements based on the speed, force and stiffness of the components, whereas the restitution model uses a simple ratio to calculate the rebound behavior. This project used the impact modeling method, as it was shown to produce good results without much tuning. The default contact values in Adams were sufficient in all criteria except the penetration depth which was increased from 0.01 to 0.001. See *Figure 5* for a screenshot of Adams UI contact definition.

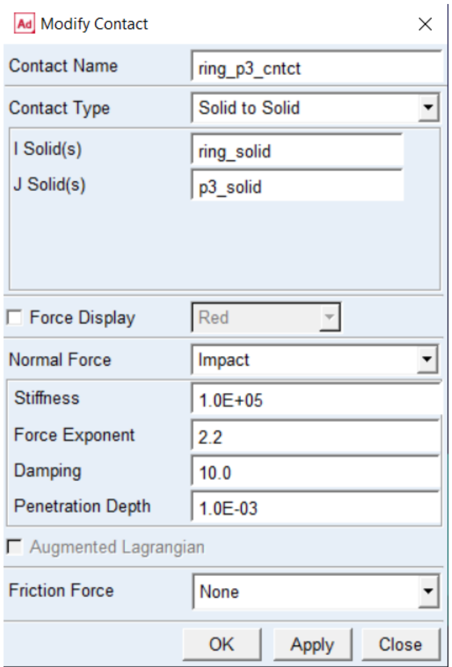


Figure 5 Contact force definition in Adams

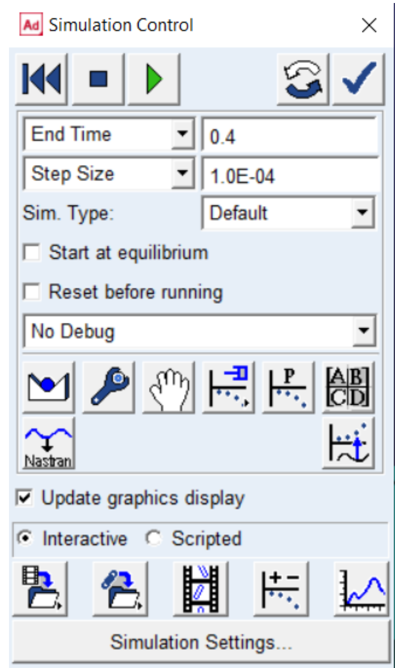


Figure 6 Simulation Setup in Adams

The setup and simulation of the contacts proved to be a significant pain point of this project, as improper contacts created wildly erroneous results. Due to the complexity of the project, it was very difficult to pin down which parameters or models were the direct cause of poor results. Initially it was assumed that there was something wrong with the designed backlash of the gears, and several different gear tooth profiles were simulated in Adams to no avail. Next the contact parameters themselves were investigated to see if they were the source of error. While tuning these parameters, one of the models was set up such that the gears would oscillate left and right, impacting the other gear teeth in rapid succession. The damping coefficient was found to be the source of the oscillation and increasing the damping ratio eliminated this type of error. After much further investigation, it was found that the sides of the planetary and ring gears were rubbing against each other during the simulation. Modifying the ring gear to have 1 millimeter of clearance between gears and the side faces solved this last issue, after which the simulations began yielding positive results.

4.3 Model Setup: Motions

The two inputs to the differential planetary gear drive were the angular velocities of the sun gear and the peripheral Z1 gear. Two different types of motion profiles were tested, a constant rotation and an exponential rotation. The constant rotation was the first motion profile to be tested and was used to validate the Adams model kinematically. This motion profile included setting the speed of the sun gear to a constant 600 degrees per second and setting the Z1 gear to a constant 300 degrees per second. Both gears were set to rotate counterclockwise in the positive Z-direction. The second motion profile applied much higher angular velocity of 6000 and 3000 degrees per second to the sun and Z1 gear respectively, also counter-clockwise in the positive Z-direction. These angular velocities were applied with an exponential function of the form (maximum speed)*(1-EXP(-time/0.05)), which allowed the gears to slowly ramp up to speed. This prevented large spikes in load at the beginning of the simulation and helped ensure smooth contact between the gears throughout the simulation. The exponential motion profiles are depicted in *Figure 7*.

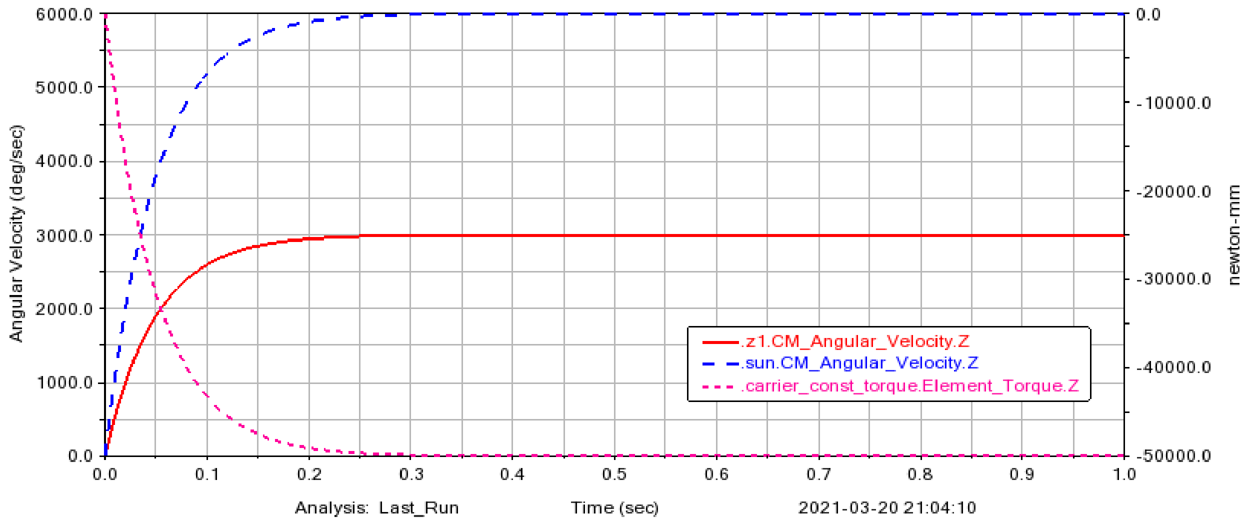


Figure 7 Model motion path and load path setup

4.4 Model Setup: Load Cases

Various different load cases were simulated throughout the course of the project, all of which involved applying a torque to the carrier which opposed the rotation of the sun and Z1 gears. The objective of this applied load was twofold: provide a contact force which could be analyzed to create FFT plots and minimize the backlash the gears experienced throughout their rotation. Without this torque, the gear contact forces saw increased noise, and the gear teeth often bounced back and forth. This load was applied in an exponential fashion with an equation of the form: (maximum torque)*(1-EXP(-time/0.05)). As will be discussed later in the report, three different magnitudes of force were applied from 0.5, 5 and 50 N-m of torque. *Figure 7* depicts the load profile of the applied torque. Note that while the torque is applied in the negative direction to counteract the positive rotation, so it appears to be decaying in *Figure 7* when it is actually growing in magnitude.

5. Validation

To validate the Adams simulation, the planetary gear train was simulated with a constant rotation on the sun gear of 600 degrees per second and a constant rotation on the Z1 gear of 300 degrees per second. Then a MATLAB script was created to calculate the respective rotations of the other components of the planetary gear train assembly. This MATLAB script is provided in Appendix B, and the calculation results are summarized in *Table 2*. The Adams simulation results for this constant angular velocity case are also presented graphically in *Figure 8*. The two results were compared by calculating a percent difference, and as shown in the table below, the two results are in good agreement with one another. This indicates that the Adams model kinematically agrees with the theoretical model.

Table 2 Comparison of simulated and analytical angular velocities

Component	Theoretical Angular Velocity (deg/sec)	Simulated Average Angular Velocity (deg/sec)	Percent Difference
Planet	-271.0425	-271.0441	0.0006%
Ring	-85.7143	-85.6484	0.0769%
Carrier	34.5865	34.6516	0.1882%

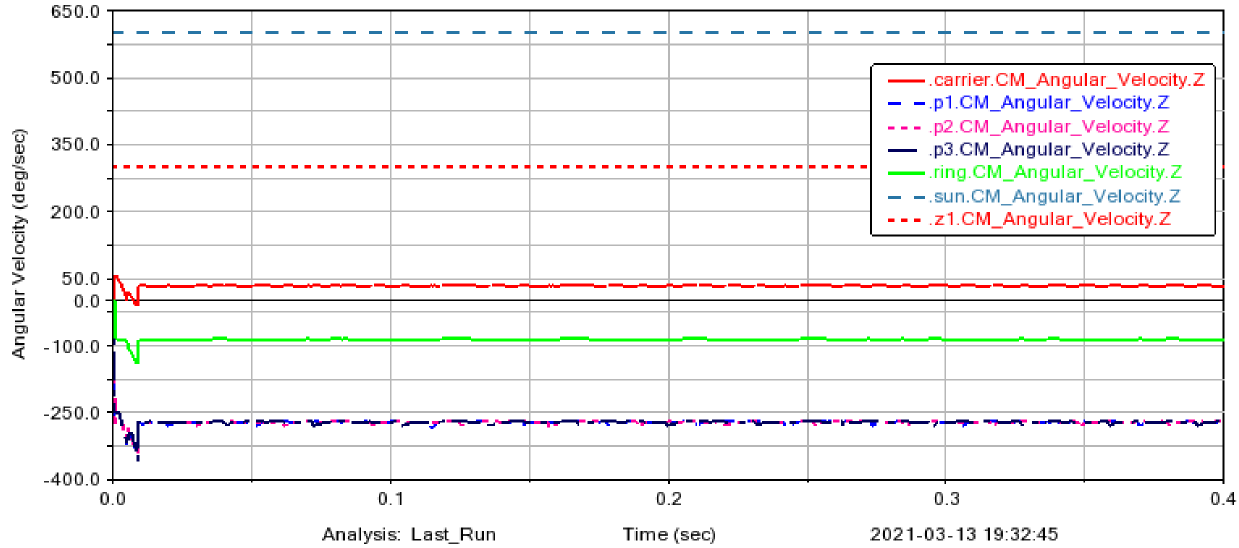


Figure 8. Adams Simulation results for the angular velocities of the planetary gear train components

6. Simulation

After constructing the Adams model joints, forces, and motions, the model was simulated using initial values provided by Dr. Wu. These simulation setup parameters included a simulation time of 0.4 seconds and a timestep of 0.001 seconds, however these settings caused the gears to have difficulty meshing and producing consistent contact forces. After further tuning, decreasing the time step to 0.0001 produced smoother contact load profiles and gear meshing. The simulation time was also increased to account for the additional time that the exponential input functions took to reach steady state.

6.2 Constant Angular Velocity Simulation Results

Simulating the planetary geartrain with constant angular velocities provided some unique insights. The most surprising result obtained was the cyclical nature of the planet gear contact forces, as presented in *Figure 9*. The figure depicts three separate gear contact forces as a function of time, one curve for each contact between the planet gears and the ring. Each gear contact force has an identical contact profile, however each curve is shifted out of phase so that the brunt of the contact load is transferred between the gears on a periodic basis. More importantly, when these individual profiles are summed together a constant force is obtained. This goes to show that the tangential forces imposed on the planet gears by the carrier are being reacted completely, albeit in a unique fashion. Furthermore, the summation of the gear forces encounters periodic oscillations at each individual peak of the contact forces. This occurs due to a new gear tooth meshing, and the speed at which the summed contact force restabilizes is controlled by the stiffness and damping constants used when defining the contact forces.

Figure 10 and *Figure 11* depict the contact load of the sun and planet gears while simulated with a constant angular velocity. One key difference between these two figures is that the introduction of a crack into the sun gear changed the profiles of the contact forces to be more erratic. This makes sense as the rough profile of the chipped tooth should have different contact area than the previously unchipped teeth. However there does not appear to be a difference between the FFT plots of the chipped and unchipped gear simulations. The gear mesh frequency for the 600 deg/sec sun gear and 300 deg/sec Z1 gear speeds was calculated to be 31.4 Hz. This frequency correlates with the second peak on the FFT plots. All remaining peaks are super harmonics of the GMF and occur at integer multiples of the GMF.

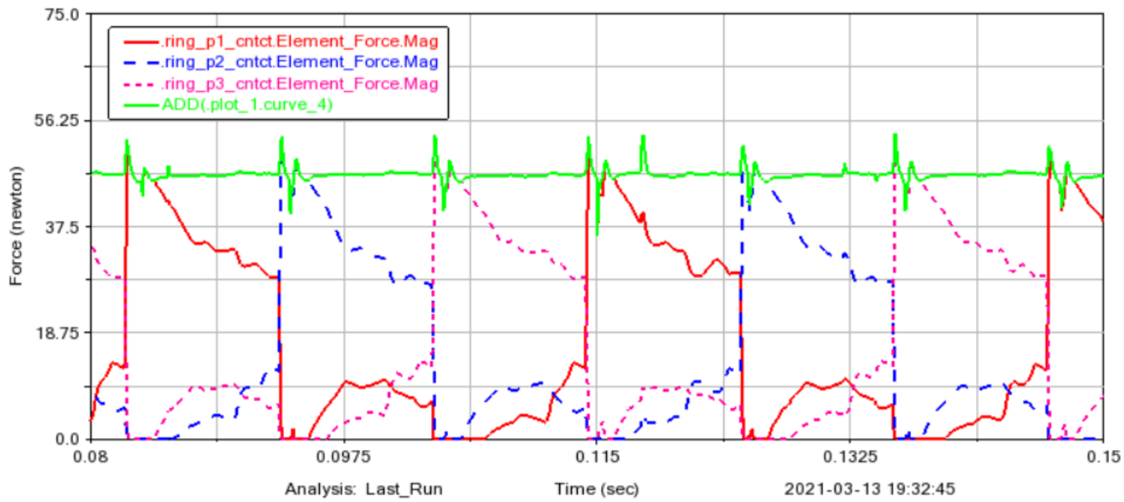


Figure 9 Ring contact force Adams simulation result and accompanying summation curve

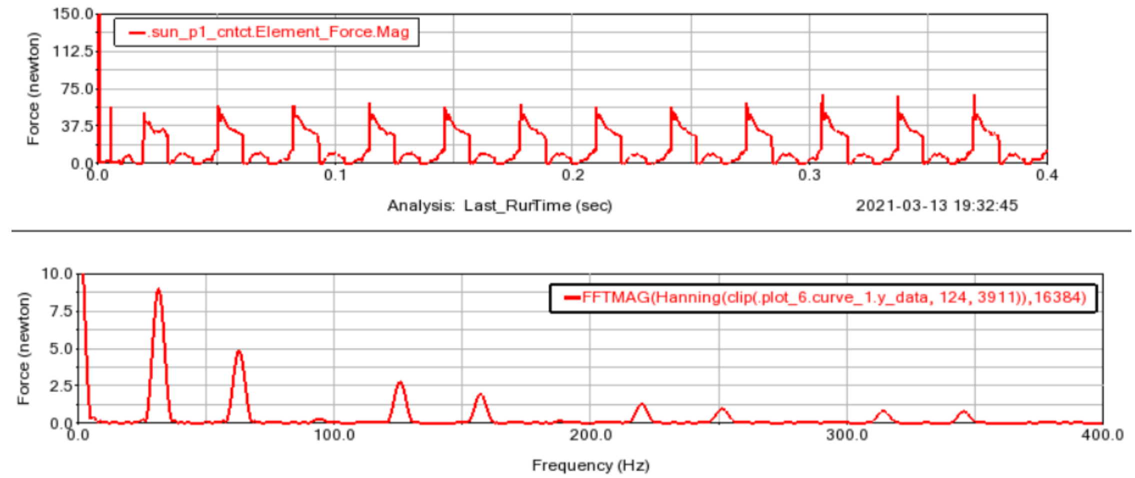


Figure 10 Uncracked geartrain contact forces and FFT plot with constant gear speeds

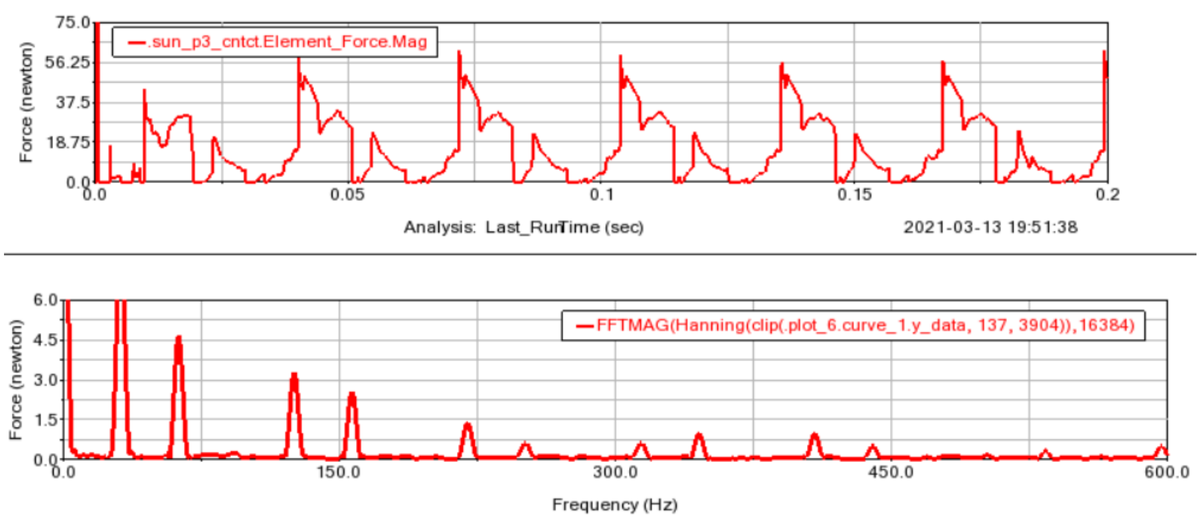


Figure 11 Cracked geartrain contact forces and FFT plot with constant gear speeds

6.3 Exponential Angular Velocity Simulation Results

After performing the simulations on the constant angular velocity load case, the exponential angular velocity load case was analyzed. The simulation setup remained the same, with the only difference being the changed load and motion functions. *Figures 12* and *13* present the FFT plots for the exponential load case. Due to the faster spin speeds of 6000 and 3000 degrees per second, the contact force plots are squished which cause them to look like a lot of noise. Zooming in on a small timespan of these contact forces yields the same profile shapes as previously shown in *Figures 10* and *Figure 11*. Unfortunately, there does not appear to be a noticeable change between the cracked and uncracked gear simulations. The GMF was calculated to be 314 Hz which lines up very closely with the first peak shown on the FFT graphs in *Figures 12-15*. When the peak frequencies were measured in Adams, they were all consistently a fraction below their theoretical GMFs. This is because the system was not at steady state for the entire duration of the FFT time window. The initially slower speeds had a lower GMF, which brought the total average down.

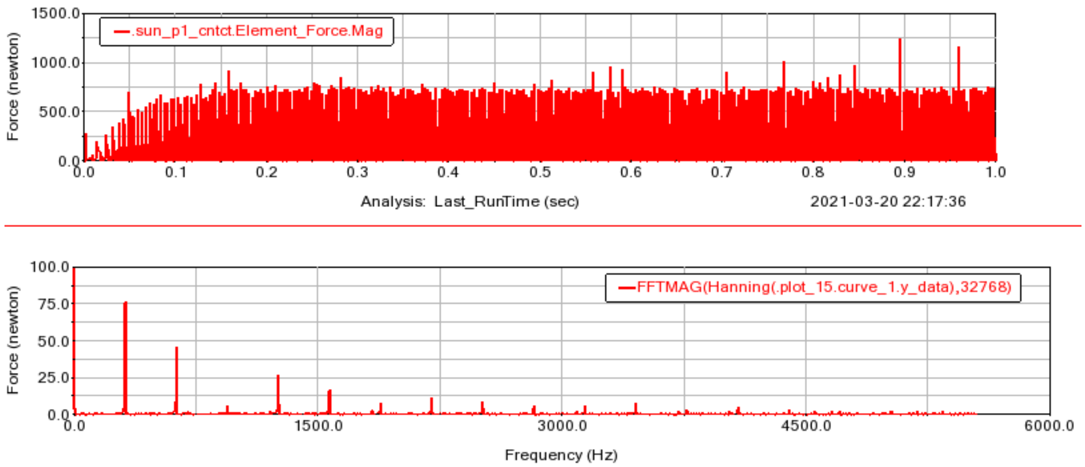


Figure 12 Cracked gearset contact force and FFT plots for exponential inputs and 50 N-m of torque

The magnitude of the input torque was simulated at several values to see its effect on the simulation results. *Figure 13* through *Figure 15* show three different torque iterations with 0.5 N-m, 5 N-m and 50 N-m loads respectively. As expected, as the magnitude of the counter torque increases, so do the magnitudes of the contact forces and FFT plots. Furthermore, only the GMF peaks of the FFT plot see significant increases in magnitude, while the amplitude of the noise between peaks did not increase. This indicates that increasing the magnitude of the counter torque can be an effective way of filtering out noise on FFT graphs.

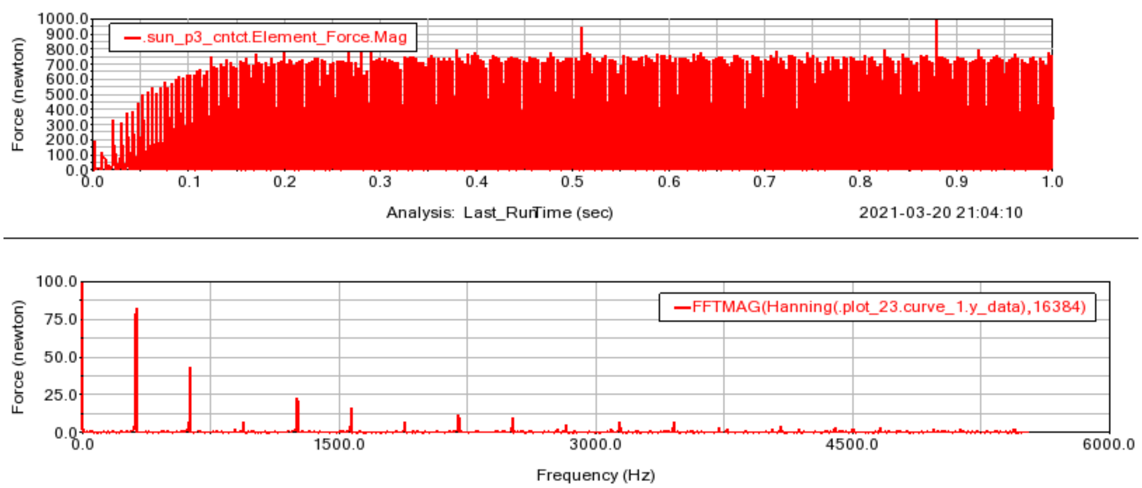


Figure 13 Uncracked gearset contact force and FFT plots for exponential inputs and 50 N-m of torque

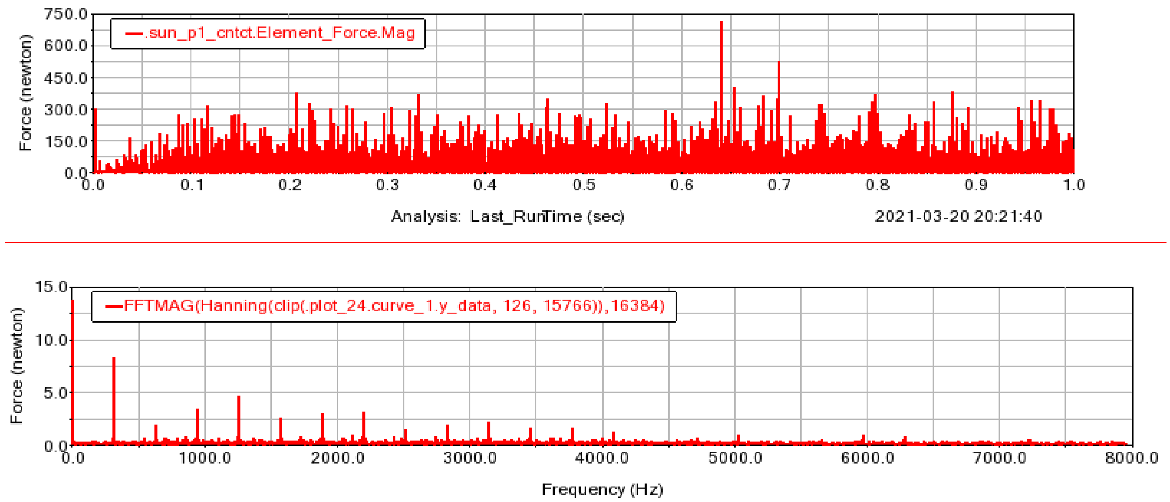


Figure 14 Cracked gearset contact force and FFT plots for exponential inputs and 5 N-m of torque

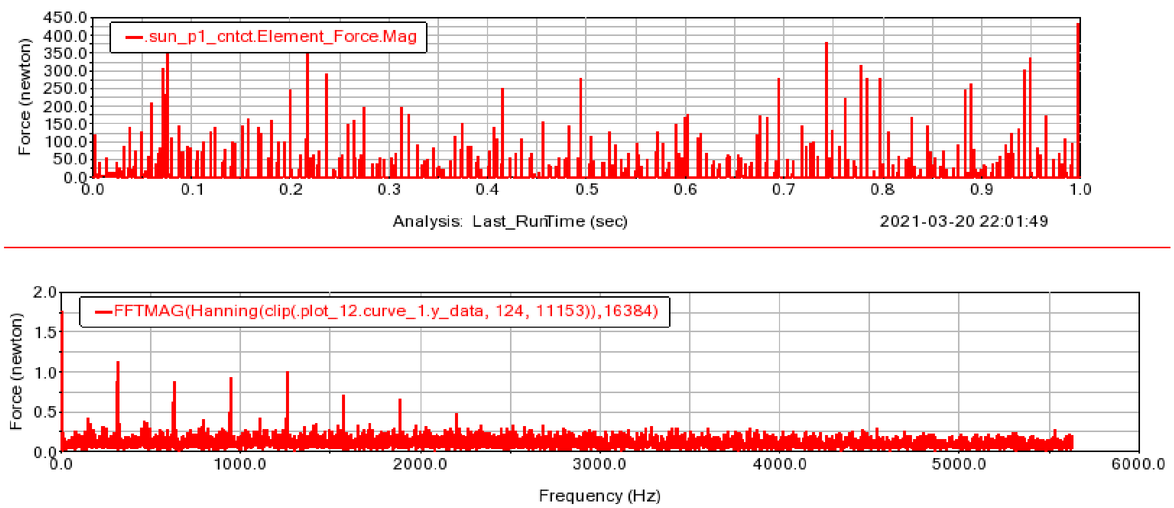


Figure 15 Cracked gearset contact force and FFT plots for exponential inputs and 0.5 N-m of torque

One final simulation was performed with new exponential functions of the form of (maximum value)*(1-exp(-time/0.3). Increasing the exponential constant from 0.05 to 0.3 causes the system to take longer to reach steady state, which causes the planetary geartrain to see angular acceleration throughout its entire simulation. A 3D FFT plot was then created to show how the gear mesh frequencies change as a function of input speeds. *Figure 16* and *Figure 18* present two different views of this 3D FFT plot while *Figure 17* depicts the theoretical calculations for the simulated motion paths. The similar shapes of the four peaks in *Figure 16* and the curves in Figure 17 indicate a strong correlation between the theoretical and simulation results.

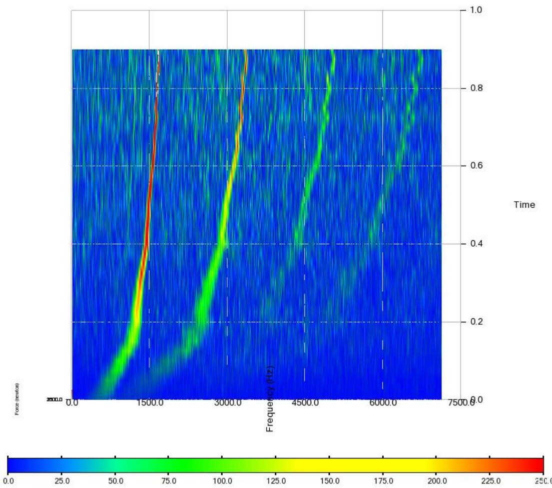


Figure 16 Adams simulation FFT plot as a function of simulation time frequencies

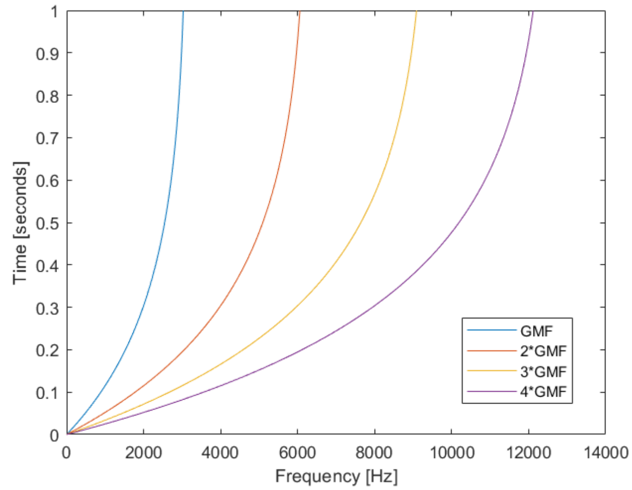


Figure 17 MATLAB calculated theoretical GCF frequencies as a function of simulation time

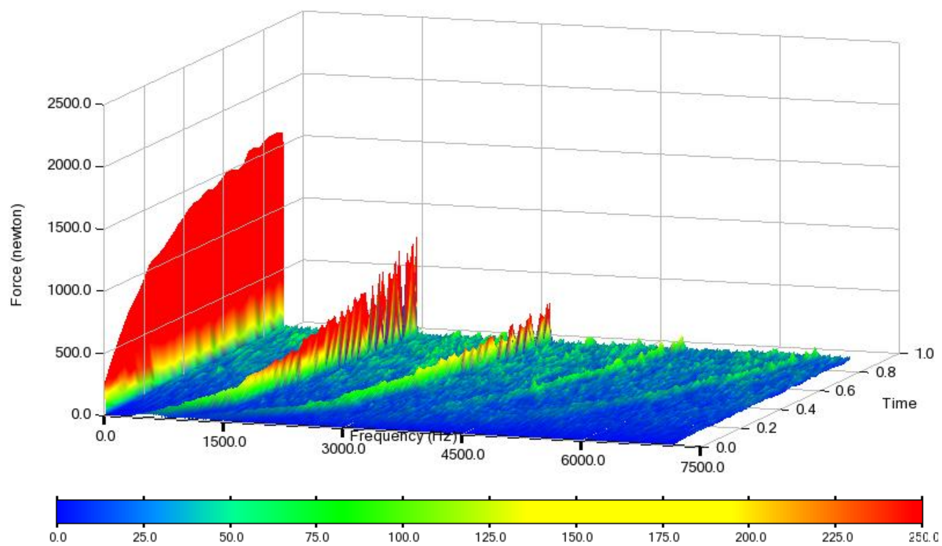


Figure 18 Isometric view a 3D FFT plot showing the gear mesh frequency as a function of input speed

Conclusions

Over the course of this project, a planetary gearset was created, assembled, simulated and analyzed through a combination of MATLAB scripts, SolidWorks CAD models and Adams simulations. Various different load cases were analyzed in Adams, including constant and exponential angular velocities. Several different counter torques were also analyzed, and increased torques were found to have noise-canceling effects on FFT plots. Finally, the gear mesh frequency progression was analyzed through a combination of Adams modeling and MATLAB simulation. Through completing this project, I have become very familiar with the Adams simulation software to the point that I would feel comfortable using it to analyze other project of mine like vehicle suspensions. I have also gained much more experience with the critical factors of both gear design and gear train analysis. Finally this project has given me a new appreciation for the complexity of the mechanical systems that surround us on a daily basis.

Table of Contents


```
clc
clear
close all
```

Variable definition

```
n_z1 = 28; % # of teeth on perimeter gear
n_z2 = 98; % # of external teeth on ring gear
n_s = 20; % # of teeth on sun gear
n_p = 37; % # of teeth on planet gears
n_r = 94; % # of internal teeth on the ring gear


m = 2; % module
rpc_z1 = m/2*n_z1; % radius of the pitch circle of gear z1
rpc_z2 = m/2*n_z2; % radius of the pitch circle of ring gear
rpc_s = m/2*n_s; % radius of the pitch circle of sun gear
rpc_p = m/2*n_p; % radius of the pitch circle of planet gears
rpc_r = m/2*n_r; % radius of the pitch circle of the ring gear

t = 0:0.01:1.0;
sun_speed = 60000.*(1-exp(-t./0.3));
Z1_speed = 30000.*(1-exp(-t./0.3));
GF_Hz = zeros(length(t),1);

for i = 1:1:length(t)
```

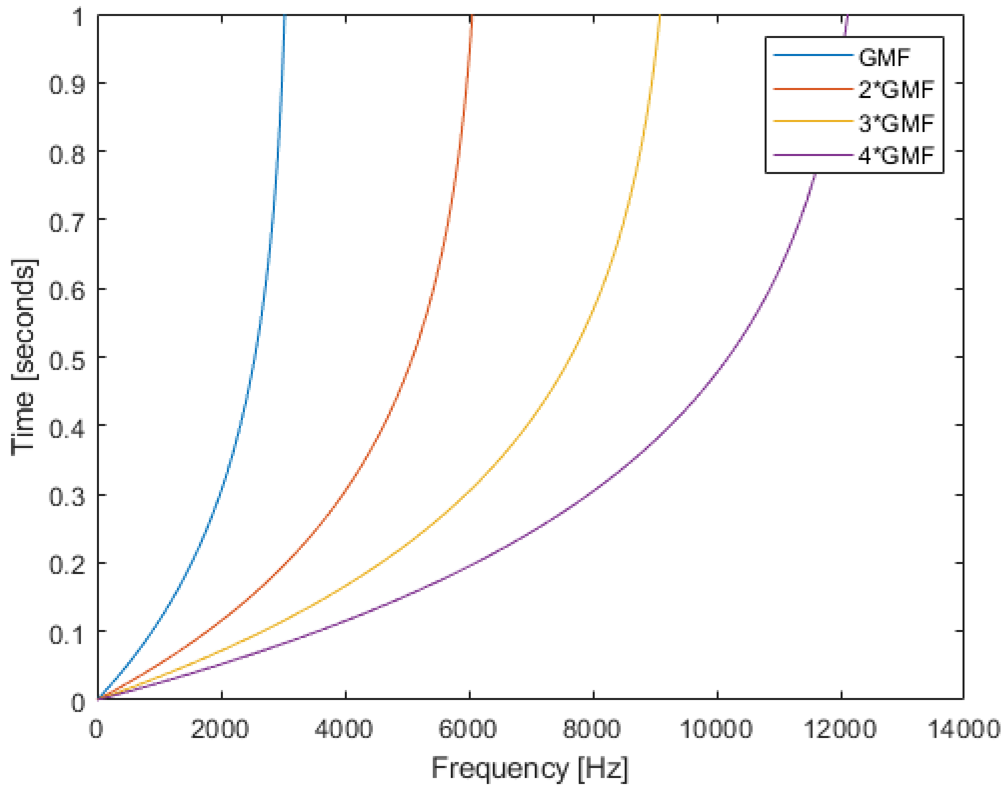
solve for gear speeds

```
syms Vz1 Vz2 Vr Vs Vc Vpi Vpo ws wp wc wr wz2 wz1

%two of the following must be known!
% Vz1 =
% Vz2 =
% Vr =
% Vs =
% Vc =
% Vpi =
% Vpo =
known1 = ws == sun_speed(i); %degrees/sec
% wp =
```

```
% wc =
% wr =
% wz2 =
known2 = wz1 == Z1_speed(i); % degrees/sec

eq1 = Vz1 == Vz2;
eq2 = Vs == Vpi;
eq3 = Vpo == Vr;
eq4 = wr == wz2;
eq5 = Vc == (Vpo-Vpi)/2;
eq6 = Vz1 == wz1*rpc_z1;
eq7 = Vz2 == -wz2*rpc_z2;
eq8 = Vs == -ws*rpc_s;
eq9 = Vr == wr*rpc_r;
eq10 = Vc == (rpc_s+rpc_p)*wc;
eq11 = Vc == wp*rpc_p+ws*rpc_s;

sol =
 solve([known1,known2,eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8,eq9,eq10,eq11],...
    [Vz1,Vz2,Vr,Vs,Vc,Vpi,Vpo,ws,wp,wc,wr,wz2,wz1]);

ws = double(sol.ws);
wp = double(sol.wp);
wc = double(sol.wc);
wr = double(sol.wr);
wz2 = double(sol.wz2);
wz1 = double(sol.wz1);
```

Calculate the gear mesh frequency

```
GF = (abs(wc-ws)*n_s); % in the same units as inputed above (degrees/
sec)
GF_rad = GF*pi()/180; % rad/s
GF_Hz(i) = GF_rad/2/pi();

end

close all
plot(GF_Hz,t,2*GF_Hz,t,3*GF_Hz,t,4*GF_Hz,t)
xlabel("Frequency [Hz]")
ylabel("Time [seconds]")
legend("GMF","2*GMF","3*GMF","4*GMF")
```



Published with MATLAB® R2020b

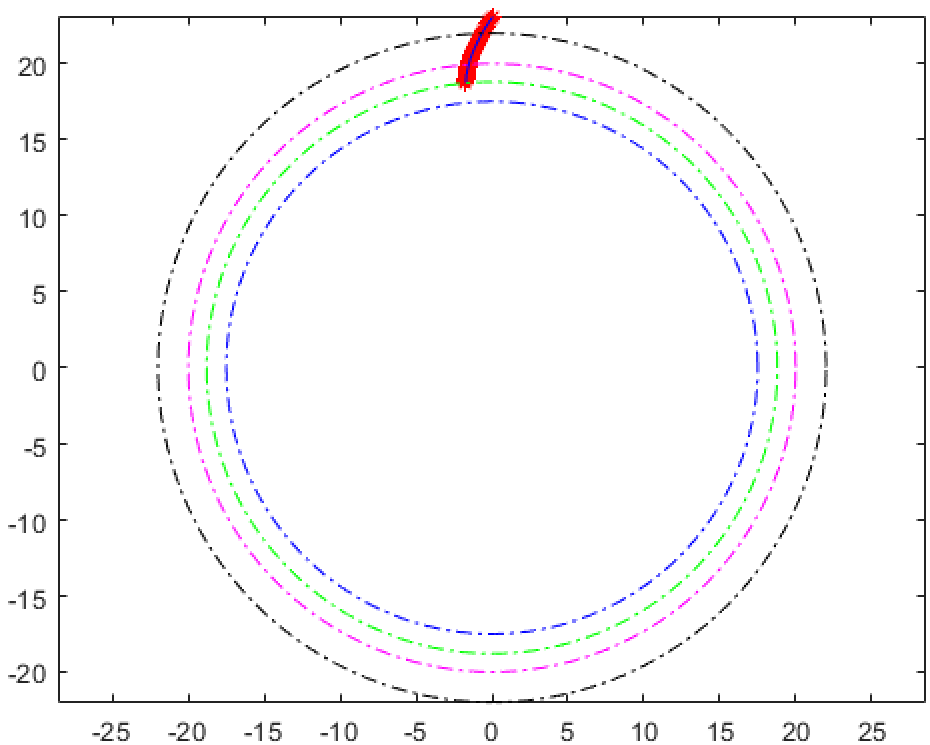
```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D.
 Luzuriaga
% DESCRIPTION: Gear parameters are specified, involute profile
 coordinates
% are sent to a tab delineated text file.

clear; close all; clc;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
filename = 'INPUT_20';                % Name of txt file for gear
type = 'EXTERNAL';                   % INTERNAL or EXTERNAL
relief = 0.98;                       % Percentage reduction in tooth
 width
                                     % to reduce interference
m = 2;                               % module (mm), or (1/Pd)
z = 20;                              % number of gear teeth
aDEG = 20;                           % pressure angle on pitch circle
 (deg)
angleRAD = pi/5;                     % This angle (rad) will determine
                                     % the length of the involute
 profile

detaA = 0.01;                        % Angular incremental step
 determines the number
                                     % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                     % addendum coeff, a constant number for
 standard gears.
cstar=0.25;                  % clearance coeff, a constant number for
 standard gears.
a=aDEG*pi/180;               % pressure angle on pitch circle (rad)

d=m*z;                       % diameter of pitch circle (mm)
da=(z+2*hstar)*m;            % diameter of addendum circle (mm)
 EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;    % diameter of dedendum circle (mm)
 EXTERNAL GEAR
db=d*cos(a);                 % diameter of base circle (mm)
s=pi*m/2;                    % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum
 circle (mm) INTERNAL GEAR
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum
 circle (mm) INTERNAL GEAR
```

```
% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on
 profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;


% Calculate half angle of external tooth thickness on base circle
 (default rot. dir. CCW)
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base
 circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth
 thickness on base circle (deg) CCW

% Calculate half angle of internal tooth thickness on base circle
 (default rot. dir. CW)
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base
 circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on
 base circle (deg) CW


% -----------------------------------------

%Rotate points
if strcmp(type,'EXTERNAL')
    rotation = AngB_O*pi/180*relief;
elseif strcmp(type,'INTERNAL')
    rotation = -AngB_I*pi/180*relief;
end
%Convert to r-theta, add rotation, then convert back to x-y
r = sqrt(x.^2 + y.^2);
theta = atan2(y,x)+ rotation;
[x,y] = pol2cart(theta,r);

% -----------------------------------------

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on
 involute profile
                                   % in matrix format (xi,yi,zi). zi =
 0
filename = [filename,'.txt'];
writematrix(gearCO,filename)
```

```
% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
Rd = dd/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
Ra = da/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n',
 AngB_O);

fprintf('\nINTERNAL GEAR\n');
Rd_I = dd_I/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
Ra_I = da_I/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n',
 AngB_I);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(1);
plot(x,y,'r*',x,y,'b-')       % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);         % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);          % pitch circle
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);         % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);         % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
```

```
axis equal
```

```
EXTERNAL GEAR
 Rd = 17.500000
 Ra = 22.000000
 Rb = 18.793852
 Rp = 20.000000

 Tooth thickness at base circle(Sb): 3.512353
 Rotation of involute from verticle(rot_EXT): 5.353958 CCW

INTERNAL GEAR
 Rd = 22.500000
 Ra = 18.755000
 Rb = 18.793852
 Rp = 20.000000

 Tooth thickness at base circle(Sb): 2.391910
 Rotation of involute from verticle(rot_INT): 3.646042 CW
```



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